NAME _____ DATE _____

5 Factoring Polynomials

5–1 *Factoring Integers*

Objective: To factor integers and to find the greatest common factor of several integers.

Vocabulary

Factor To write a number as a product of numbers. For example, $72 = 8 \cdot 9$.

Factor set The set over which a number is factored.

Prime number, or prime An integer greater than 1 that has no positive integral factor other than itself and 1. For example, 19 is prime.

Prime factorization Writing a positive integer as a product of primes. For example, the prime factorization of 30 is $2 \cdot 3 \cdot 5$.

Common factor A factor of two or more integers is called a common factor of the integers. For example, 3 is a common factor of 6 and 9.

Greatest common factor (GCF) The greatest integer that is a factor of two or more given integers.

Example 1 List all the positive factors of 42.

Solution

$42 = 1 \cdot 42$
$= 2 \cdot 21$
$= 3 \cdot 14$
$= 6 \cdot 7$
$(= 7 \cdot 6)$

Divide 42 by 1, 2, 3, ... until a pair of factors is repeated.

The positive factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

List all of the positive factors of each number.

1. 10 **1,2,5,10**
2. 24 **1,2,3,4,6,8,12,24**
3. 36 **1,2,3,4,6,9, 12,18,36**
4. 40 **1,2,4,5,8,10,20,40**
5. 17 **1,17**
6. 54 **1,2,3,6,9,18, 27,54**
7. 29 **1,29**
8. 42 **1,2,3,6,7,14, 21,42**

CAUTION Factors come in pairs. For example, since $12 \div 3 = 4$, 3 and 4 are both factors of 12.

Example 2 List all pairs of factors of each integer: **a.** 18 **b.** −18

Solution

a. $(1)(18)$ $(-1)(-18)$
$(2)(9)$ $(-2)(-9)$
$(3)(6)$ $(-3)(-6)$

b. $(1)(-18)$ $(-1)(18)$
$(2)(-9)$ $(-2)(9)$
$(3)(-6)$ $(-3)(6)$

List all pairs of factors of each integer. Answers given at the back of this Answer Key.

9. 11
10. 20
11. 23
12. 39
13. 57
14. 60
15. 75
16. 78
17. 81
18. 105
19. 121
20. −30
21. −63
22. −57
23. −93

5–1 *Factoring Integers* (continued)

Example 3 Find the prime factorization of 252.

Solution Try the primes in order as divisors. Divide by each prime as many times as possible before going on to the next prime. Stop when all factors are primes.

$252 = 2 \cdot 126$
$= 2 \cdot 2 \cdot 63$
$= 2 \cdot 2 \cdot 3 \cdot 21$
$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$
$= 2^2 \cdot 3^2 \cdot 7$

Find the prime factorization of each number. A calculator may be helpful.

24. 22 **$2 \cdot 11$**
25. 30 **$2 \cdot 3 \cdot 5$**
26. 56 **$2^3 \cdot 7$**
27. 64 **2^6**
28. 44 **$2^2 \cdot 11$**
29. 50 **$2 \cdot 5^2$**
30. 72 **$2^3 \cdot 3^2$**
31. 84 **$2^2 \cdot 3 \cdot 7$**
32. 93 **$3 \cdot 31$**
33. 180 **$2^2 \cdot 3^2 \cdot 5$**
34. 275 **$5^2 \cdot 11$**
35. 388 **$2^2 \cdot 97$**

Example 4 Find the GCF of 540 and 264.

Solution

1. First find the prime factorization of each integer.
   $540 = 2^2 \cdot 3^3 \cdot 5$ $264 = 2^3 \cdot 3 \cdot 11$
2. Then find the product of smaller powers of each common prime factor.
   The common prime factors are 2 and 3.
   The smaller power of 2 is 2^2.
   The smaller power of 3 is 3.
3. The GCF of 540 and 264 is $2^2 \cdot 3$ or 12.

CAUTION If there are no common prime factors, the GCF is 1. For example, since $12 = 2^2 \cdot 3$ and $25 = 5^2$, the GCF of 12 and 25 is 1.

Find the GCF of each group of numbers. A calculator may be helpful.

36. 36, 90 **18**
37. 28, 70 **14**
38. 120, 128 **8**
39. 108, 180 **36**
40. 105, 350 **35**
41. 126, 144 **18**
42. 145, 174 **29**
43. 260, 325 **65**

Mixed Review Exercises

Simplify.

1. $\frac{1}{2}(4x + 2) + 3\left(\frac{1}{3}x - 1\right)$ **$3x - 2$**
2. $(4 + 3)^2$ **49**
3. $2^2 + (3 + 1)^2$ **20**
4. $2x - 3 - (2x + 4)$ **-7**
5. $2ab(3a^2)(4b)$ **$24a^3b^2$**
6. $2x^3(3y)(5y)$ **$30x^3y^2$**
7. $(2x)^3$ **$8x^3$**
8. $3n(2n^2 - 5n) + 7n^2$ **$6n^3 - 8n^2$**
9. $(-3)^4x^4$ **$81x^4$**
10. $x(x^2 - 2) - x^2(x + 4)$ **$-2x - 4x^2$**
11. $(3y + 4)(y + 2)$ **$3y^2 + 10y + 8$**
12. $(x - 3)(2x + 3)$ **$2x^2 - 3x - 9$**